Economic Analysis of the Reasonable Royalty: Simplification and Extension of the Georgia-Pacific Factors

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1. INTRODUCTION

Damages in most patent infringement lawsuits hinge on determining a "reasonable royalty" that the infringer would be willing to pay and still be able to make a reasonable profit. The fifteen "Georgia-Pacific factors" have served as the standard framework for this purpose for over 30 years, with all of the limitations identified in Judge Tenney's decision.[1] The overriding problem is that the factors typically can be used to justify a very wide range of outcomes. To cope with the practical need to arrive at an appropriate royalty, influential decisions since *Georgia-Pacific* have oscillated between using more explicit economic analysis, in some cases, to using no formal economic analysis in others.

What has been lacking is a simple, coherent method for updating the insightful legal framework established by *Georgia-Pacific* with modern principles of corporate finance. These principles reflect the "best practices" of academic research and real-world investment analysis for quantifying a reasonable profit for the infringer and for analyzing the size of a royalty that is consistent with such a profit level. This article makes that connection by putting forward a new royalty approach denoted as FIRMM – for Financial Indicative Running Royalty Model. FIRRM is a powerful method for reducing the uncertainty regarding a reasonable

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[1] Georgia-Pacific Corp. v. United States Plywood Corp., 318 F.Supp. 1116, 1120, 1121.

royalty and for achieving more reliable awards with less time and expense.

In addition, FIRRM provides an economically rational alternative to the 25% (of profits) and 5% (of revenue) rules, which are sometimes used to calculate a royalty. These rules are arbitrary and, in most cases, mutually inconsistent. FIRRM should also be superior to royalties from surveys when, as often appears to be the case, the deals that are surveyed fail to be sufficiently comparable to the case at hand.

The original Georgia-Pacific case reveals the scope of the potential contribution from improved tools for finding a reasonable royalty. After more than four years of hearings (where both sides agreed upon the relevant "doctrinal criteria"), Georgia-Pacific still argued that the royalty should be under 2% of the infringing revenue, while the patent holder U.S. Plywood claimed a minimum royalty of approximately 31%.[2] Judge Tenney awarded the higher rate but the case did not end there. The Court of Appeals found that this rate "did not allow GP a reasonable profit after paying the suppositious royalty…a basic error" and finally awarded a rate of about 22%.[3]

Extremely divergent claims have continued in cases up to the present, resulting in the widely cited appellate decision by the Federal Circuit in *SmithKline Diagnostics v. Helena* that a court "may reject the extreme figures proffered by the litigants as incredible and substitute an intermediate figure as a matter of its judgment."[4] FIRRM offers an objective analysis for these situations that can yield a high, low, or intermediate royalty depending on the data of a particular case.

This article is organized as follows. Section 2 offers a brief, non-technical review of the basic principles of economics and investment analysis that should underlie a reasonable royalty award. Section 3 describes the FIRMM model and its application in arriving at such awards. Section 4 shows how FIRMM relates to the existing legal and economic framework, operating consistently with the Georgia-Pacific and subsequent decisions, while avoiding the pitfalls sometimes encountered in royalty litigation. Section 5 uses several actual cases to illustrate how FIRRM would be applied in practice. Section 6 contains

[2] *Georgia-Pacific*, 318 F.Supp. 1116, 1119.

[3] Georgia-Pacific Corp. v. U.S. Plywood-Champion Papers Inc., 446 F.2d 299; 170 U.S.P.Q. (BNA) 369.

[4] SmithKline Diagnostics, Inc. v. Helena Laboratories Corp., 926 F.2d 1168.

some brief concluding remarks. The mathematical description of FIRRM is relegated to the Appendix.

2. ECONOMIC PRINCIPLES FOR A REASONABLE ROYALTY

A. ALTERNATIVE PROJECTS, PROFITS, AND THE COST OF CAPITAL

A reasonable royalty is frequently defined in the cases as an amount "which a person, desiring to use a patented article, as a business proposition, would be willing to pay as a royalty and yet be able to use the patented article at a reasonable profit."**5** The patent statute does not itself specify a methodology for this purpose, stating only that

> Upon finding for the claimant the court shall award the claimant damages adequate to compensate for the infringement, but in no event less than a reasonable royalty for the use made of the invention by the infringer, together with interest and costs as fixed by the court.[6]

In other words, a reasonable royalty is intended to be the *minimum* damages award.[7] The reasonable royalty is usually interpreted as the amount an infringer would have been willing to pay for the patented technology in a hypothetical business negotiation. The infringer's willingness to pay for the patented technology is a sensible basis for the royalty and the principles of corporate finance and investment analysis provide a coherent framework for making this determination.

The key is to compare the profits from the infringing activity to the profits from the infringer's "next-best" alternative project at the time of the hypothetical negotiation. That is, the infringing activity is viewed as an investment, typically involving the intellectual property ("IP") at issue coupled with expenditure on complementary investments in plant and equipment, working capital, and other IP assets, with the purpose of earning the largest possible profit. The maximum willingness to pay for the relevant patent rights would then depend on the profitability of the

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[5] See, among others, Panduit Corp. v. Stahlin Bros. Fibre Works, Inc., 575 F.2d 1152, 1158.

[6] 35 U.S.C. § 284.

[7] The patent statute provides for punitive damages in certain circumstances, as well as a possible award of lost profits.

alternative project as a "replacement" for the infringing activity.[8] An alternative project that was nearly as profitable implies that a negotiation would result in only a low royalty. Lower value alternatives imply higher reasonable royalties. Whatever the best alternative would be, it establishes the profit the infringer could have expected to earn absent the infringement.

The alternative project can take a variety of forms. For example, the infringer might produce a similar product by licensing alternative technology. The infringer may also be able to produce a non-infringing substitute by designing around the patent. The infringer could also postpone the investment until after patent expiration. In general, it is necessary only to consider alternative investments that can substitute for the patented article or process.[9]

The difference in profits between the infringing project and the alternative is the basis for the reasonable royalty developed in this article. For example, suppose a firm could engage in an infringing investment project that is expected to produce profits of $10 million. The next best alternative does not infringe on the patent, but would yield profits of only $9 million. In a hypothetical royalty negotiation, the infringer would be willing to pay a royalty of at most $1 million (either as a single up-front payment or as a running royalty on future sales with an equivalent present value). The ability of the patent holder to extract value through the royalty is limited by the value of the infringer's alternative investment.

The infringer's willingness to pay when there is no alternative to the infringing project is bounded by the need to earn a minimum return, which is required in order for any investment to be worthwhile. This minimum return is referred to in investment analysis as the project's "cost of capital," the rate demanded by the investors who are the source of financing for projects. The project must have a return at least equal to the cost of capital to be competitive or investors will commit their capital elsewhere.[10] The reasonable royalty, absent punitive damages, should be

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[8] Compare Marcus Finnegan and Herbert Mintz, "The Determination of Reasonable Royalty" in The Law and Business of Patent and Know-How Licensing 6th edition, edited by Brian G. Brunsvold and Dennis P. O'Reilley at I-8: "The maximum royalty that would normally be reasonable for a licensee to pay is that which is less than the cost of the next best alternative available to the licensee."

[9] In the absence of financial or other constraints, the infringer presumably is already undertaking all other profitable projects.

[10] For a detailed discussion of the calculation of the cost of capital and related issues see Richard A. Brealey, Stewart C. Myers, and Alan J. Marcus, *Fundamentals of Corporate Finance* 2001 at 317.

no higher than the level that is expected to allow the infringer to earn the cost of capital or the hypothetical royalty negotiation would not be successful.

The cost of capital needs to be earned on the project as a whole. In particular, the calculation of the expected return should take into account all incremental expenses and capital investment needed to commercialize the patent at issue. Incremental costs are not limited to direct costs for labor and materials. For example, costly specialized production facilities or a requirement for an expensive marketing campaign for the new product would lower the return and the royalty that could be paid.

The reasonable royalty calculated on this basis may be higher or lower than the patent holder's lost profits from the infringement. Lost profits may include damages from lost sales and damages from price erosion. Since the patent holder is entitled to lost profits if they exceed the royalty, a separate analysis of lost profits is typically required in order to determine which form of compensation is most appropriate.[11]

B. NET PRESENT VALUE AND THE INTERNAL RATE OF RETURN

At bottom, investment projects are evaluated in terms of the cash they are expected to generate. Because projects differ in the timing and risk of their cash flows, it is necessary to discount the returns to present value in order to compare them meaningfully. The net present value (NPV) of a project is the present value of the expected future cash flows, discounted at the cost of capital, net of the amount of any initial investment. The concept of NPV underlies all analysis of investment projects.[12]

The timing of the cash flows is critical when measuring profitability. For example, suppose that project consists of an up-front expenditure of $100 and that it is expected to generate a single cash flow of $120. Assume also that the cost of capital is 10%. If the return is expected in one year then the cash flow would have a *present* value of only $109.09 ($120 divided by 1 + 10%, or 1.10). The NPV of the project would

[11] For a recent economic analysis of lost profits see Roy J. Epstein, The Market Share Rule with Price Erosion: Patent Infringement Lost Profits Damages after Crystal, 31 AIPLA Q.J. 1 (2003).

[12] For a detailed discussion of NPV see Brealey, Myers, and Marcus at 163. A discount rate is used like a compound interest rate in calculating NPV. Higher or lower risk of the project's cash flows implies a correspondingly higher or lower discount rate.

accordingly be $9.09 after subtracting the value of the initial investment. If the return were not expected until three years in the future, then its present value would fall to $90.16 ($120 divided by $(1 + 10\%)^3$, i.e., 1.10^3). In this case the project NPV would be *negative* $9.84. The fact that the project ultimately returns $20 above its cost is not sufficient to determine true economic profitability.

A positive NPV means that the project is worth more than its cost and would be worthwhile for the firm. Negative NPV projects destroy value and should not be undertaken. The requirement that investors earn their cost of capital from a project is equivalent to saying that viable projects should have a positive NPV (investors would be indifferent about zero NPV projects).

The final investment concept used in the royalty analysis is known as the "internal rate of return" (IRR). The IRR can be understood as the effective rate of interest rate earned on the investment, irrespective of the cost of capital. It is defined as the value of the discount rate that yields an NPV of zero for the project. Consider the example where an investment of $100 would return $120 in one year. The internal rate of return on the project would then be 20%, since $120 discounted at this rate equals the $100 investment. It is an "internal" rate in the sense that it is determined only by the project's own cash flows (unlike the cost of capital, which is a competitive rate of return determined outside the firm in the capital market).

The IRR can be compared to the cost of capital to indicate project profitability. This is particularly useful in a royalty analysis because the documents in the litigation often already provide information on the cost of capital and the IRR of the infringing project. Specifically, we call the amount by which the IRR exceeds the cost of capital the "IRR spread," or simply "spread" when the context is clear. A positive (negative) spread implies a positive (negative) NPV, so the spread is useful to compare the relative value of projects. For example, if the firm has a cost of capital equal to 10%, a project with a 15% IRR has a spread of 5%. A spread of zero means the project has zero net present value because the IRR is equal to the cost of capital. A project will support a royalty payment only when the spread is positive, otherwise it would not be possible for the project to earn its cost of capital.

3. THE FINANCIAL INDICATIVE RUNNING ROYALTY MODEL

FIRRM determines the maximum running royalty that would be competitive in a hypothetical negotiation. This running royalty has a value equal to the difference between the NPV of the infringing project and the NPV of the infringer's next-best alternative. This approach assumes that the FIRRM royalty would also be acceptable to the patent holder. There can be cases where this assumption is not justified, specifically when the patent holder could prove that the expected losses from the infringement would exceed the royalty income.[13] However, in this situation a lost profits standard, not a reasonable royalty standard, seems more relevant, so there would be less interest in a FIRRM-style solution.

FIRRM is an indicative model because it is based on a simplified cash flow scenario. The approach has the practical advantages of reasonable accuracy, flexibility, and low cost of implementation. In an actual application, FIRRM could be used in conjunction with detailed, traditional discounted cash flow ("DCF") analysis of the infringing project.[14] In fact, FIRRM can approximate DCF analysis to any desired degree of accuracy. It is easy to test the sensitivity of the results to changes in the values of the key assumptions and to incorporate progressively greater levels of detail in the analysis. FIRRM can be especially convenient because the documents in infringement litigation often already provide information on the key parameters required to calibrate the model.

A. BASIC ANALYSIS

We start with a basic model in which the infringing project requires a single up-front investment and then returns a stream of constant cash flows. This pattern should represent many investments

[13] In this case, there could not be a successful hypothetical negotiation between the patent holder and (potential) infringer. The minimum acceptable price of the patent holder would be greater than the maximum viable offer of the infringer, and there would be no mutually acceptable royalty.

[14] DCF analysis for determination of a reasonable royalty is advocated by a growing number of writers. See, for example, Russell Parr, *Intellectual Property Infringement Damages* at 123; Daniel Burns, "DCF Analyses in Determining Royalty," *les Nouvelles* (September, 1995) at 165; and Lauren J. Stiroh and Richard T. Rapp, "Modern Methods for the Valuation of Intellectual Property," available online at www.nera.com.

reasonably well because it essentially captures the initial investment and the "steady state" terminal values in standard cash flow investment models.

For a flavor of the approach, consider a simple example where the infringing project lasts two periods. In the first period there are no sales and an initial investment of $100 million. In period two the project is expected to generate a positive cash flow of $125 million on sales of $250 million (implying a 50% gross profit margin). The appropriate discount rate is given by the cost of capital, which is assumed to be 15%. These assumptions imply that project NPV is $8.7 million (i.e., –100 + 125/1.15). The IRR is 25%. Suppose the non-infringing alternative has an NPV equal to 25% of the infringing project, or $2.175 million.

The maximum competitive lump-sum (i.e., paid-in-advance) royalty would be $6.525 million, or the difference in NPV between the infringing project and the alternative investment ($8.7 million minus $2.175 million). If structured as a running royalty (i.e., as an annual, end-of-year fee), the maximum competitive royalty would be $7.5 million, also yielding a present value of $6.525 million ($7.5 million/1.15, rounded). The running royalty is 3% of revenue and 6% of profits (7.5/250 and 7.5/125, respectively). The analysis is easily generalized to more realistic cases. While the calculations become somewhat more complex, the principle is fixed: find the running royalty that makes the potential infringer just indifferent between paying for the patented technology, or moving on to the next-best alternative.

In general, FIRRM determines the running royalty as a function of four parameters of the infringing project: the cost of capital, the IRR spread, useful economic life, and the ratio of the NPV of the alternative to the NPV of the infringing project (which measures the ability of the alternative to "replace" the infringing profits). The mathematical details of the model, based on annuities, are presented in the Appendix. In the example just cited, the spread is 10%, the useful life is one year, and the NPV ratio is 25%. The model returns a value of 6% for the royalty as a percentage of profits. With a 50% profit margin, the corresponding royalty as a percentage of revenue would be 3% (6% times 50%).

Table 1 illustrates FIRRM running royalties as a percent of profit for various configurations of the key parameters. In each scenario, the cost of capital is assumed to be 15% and the IRR spreads range from 0%

to 100%.[15] Panel a shows results for a useful life of 5 years and an NPV replacement ratio of 0%. Panel b illustrates a 5 year useful life and an NPV replacement ratio of 50%. Panels c and d contain corresponding results for a 10 year useful life. It is straightforward to evaluate the model for alternative input assumptions.

TABLE 1

FIRRM Indicative Royalties
(% of Infringer's Pre-Tax Incremental Profit)

5 Year Life

a) 0% NPV Replacement Ratio b) 50.0% NPV Replacement Ratio

IRR Spread	Royalty	IRR Spread	Royalty
0%	0.0%	0%	0.0%
5%	10.8%	5%	5.4%
10%	19.8%	10%	9.9%
15%	27.3%	15%	13.7%
20%	33.8%	20%	16.9%
25%	39.3%	25%	19.6%
100%	74.6%	100%	37.3%

10 Year Life

c) 0% NPV Replacement Ratio d) 50.0% NPV Replacement Ratio

IRR Spread	Royalty	IRR Spread	Royalty
0%	0.0%	0%	0.0%
5%	16.5%	5%	8.2%
10%	28.9%	10%	14.4%
15%	38.4%	15%	19.2%
20%	45.9%	20%	23.0%

[15] For values of the IRR spread exceeding 100% (typically corresponding to small up-front investment costs), the FIRRM royalty rate on profits approaches 100% when there are no patent expiration effects. In this situation even a large change in the spread has only a modest effect on the royalty. For example, Table 1c shows a royalty rate of 82.7% for a 100% spread. Doubling the spread to 200% would raise the royalty to only 90.7%.

25%	51.9%	25%	26.0%
100%	82.7%	100%	41.3%

Note: based on 15% cost of capital

The important lesson conveyed by Table 1 is that it can be highly misleading to focus on a single parameter or on an apparently reasonable rule of thumb as justification for a particular royalty amount. For example, the reasonable royalty on profits for a 5-year investment and a zero-NPV alternative varies from 0% to almost 75%, depending on the IRR spread. The useful life also has a nonlinear effect. The royalty for a 10-year investment with a zero NPV ratio exceeds the corresponding 5-year royalty by about 5.7 percentage points at a 5% IRR spread but the gap rises to 12.1% percentage points with a 20% spread and then ultimately approaches zero for very large IRR spreads. The royalty scales with the NPV ratio, so a 50% replacement ratio cuts the maximum negotiated royalty in half. Finally, increases in the cost of capital lower the royalty (through reducing the effective spread), but this effect is not shown in the table.

The model thus shows that the economic facts in a given case can result in royalties as low as zero and nearly as high as the entire profit from the project. There is a high degree of interaction among the different factors; this highlights the importance of using the model to quantify the different effects reliably. Rules of thumb for a royalty such as 25% of profit or 5% of revenue will be appropriate only by happenstance; more likely, they will differ substantially from the appropriate values.

Table 2 restates the royalties in terms of percentages of the infringer's revenue. The royalty as a percent of revenue equals the corresponding royalty on profit multiplied by the gross profit margin. Consider the 5-year investment with a zero-NPV alternative and a spread of 10%. The royalty on profit in Table 1 is 19.8%. The royalty on revenue is 4% when the profit margin is 20% (19.8% times 20%) but rises to 9.9% if the profit margin is 50% (19.8% times 50%).

FIRRM makes it clear that a high gross profit margin does not automatically support a high royalty on revenue. Recall that a high profit margin may still be associated with a project with a low spread when the initial investment is large relative to later cash flows. The low spread would imply a relatively small royalty. For example, the royalty

corresponding to the 50% profit rate just discussed in Table 2 falls from 9.9% to 5.4% if the IRR spread is only 5%. In general, the values in Table 2 depend on the underlying parameters in the same way as they do in Table 1.

TABLE 2

FIRRM Indicative Royalties
(% of Infringer's Revenues)

5 Year Life

a) 0% NPV Replacement Ratio

IRR	Royalty with Infringer's Incremental Profit Margin @			
Spread	20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	2.2%	3.2%	4.3%	5.4%
10%	4.0%	5.9%	7.9%	9.9%
15%	5.5%	8.2%	10.9%	13.7%
20%	6.8%	10.1%	13.5%	16.9%
25%	7.9%	11.8%	15.7%	19.6%
100%	14.9%	22.4%	29.8%	37.3%

b) 50.0% NPV Replacement Ratio

IRR	Royalty with Infringer's Incremental Profit Margin @			
Spread	20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	1.1%	1.6%	2.2%	2.7%
10%	2.0%	3.0%	4.0%	4.9%
15%	2.7%	4.1%	5.5%	6.8%
20%	3.4%	5.1%	6.8%	8.4%
25%	3.9%	5.9%	7.9%	9.8%
100%	7.5%	11.2%	14.9%	18.7%

10 Year Life

c) 0% NPV Replacement Ratio

IRR	Royalty with Infringer's Incremental Profit Margin @			
Spread	20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	3.3%	4.9%	6.6%	8.2%
10%	5.8%	8.7%	11.5%	14.4%
15%	7.7%	11.5%	15.4%	19.2%
20%	9.2%	13.8%	18.4%	23.0%
25%	10.4%	15.6%	20.8%	26.0%
100%	16.5%	24.8%	33.1%	41.3%

d) 50.0% NPV Replacement Ratio

IRR	Royalty with Infringer's Incremental Profit Margin @			
Spread	20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	1.6%	2.5%	3.3%	4.1%
10%	2.9%	4.3%	5.8%	7.2%
15%	3.8%	5.8%	7.7%	9.6%
20%	4.6%	6.9%	9.2%	11.5%
25%	5.2%	7.8%	10.4%	13.0%
100%	8.3%	12.4%	16.5%	20.7%

Note: based on 15% cost of capital

The range of royalties in Tables 1 and 2 are useful indicators of what to expect from more detailed analyses of time-varying cash flows. The tables indicate the maximum viable royalty from the point of view of the infringer, which is not necessarily the level that would emerge from a hypothetical negotiation with an anxious licensor. However, the cases often appear to set damages based on a maximum reasonable royalty since the "setting of a reasonable royalty after infringement cannot be treated…as the equivalent of ordinary royalty negotiations among truly 'willing' patent owners."[16] FIRRM is consistent with this principle of setting higher "ex post" royalties and it has the advantage of ensuring that the award still meets the requirement of profitability for the infringer, a result that does not always follow when using other approaches.

B. FIRRM WITH PATENT EXPIRATION

FIRRM is easily extended to the important situation of an investment that is initially infringing but lasts beyond the expiration of the patent. The cash flows after patent expiration are assumed to be immune from infringement liability. In this setting, the infringing project initially generates a high cash flow during the patent period, followed by a smaller cash flow after patent expiration (due to entry and competition by other firms). As before, the maximum royalty is the difference in NPV between the infringing project and the alternative. See the Appendix. For simplicity, it will be assumed that the royalty can be assessed only on cash flows accruing during the patent period.[17]

Patent expiration creates another important issue for the reasonable royalty. The infringer is likely to have an economically meaningful option to postpone the project until after patent expiration, especially when the patent is due to expire in the near future. This option can provide a significant replacement ratio even when there is no alternative to the patented technology. Moreover, the ratio should increase as the postponement is shortened, implying a lower present value for the

[16] See *Panduit*, 575 F.2d 1152, 1158.

[17] Since FIRRM takes account of the present value (as of the time of the hypothetical negotiation) of the cash flows extending beyond the patent period, the model is able to reflect damages arising from accelerated entry.

reasonable royalty. This conclusion should be intuitively obvious, but such intuition is not always reflected in the decided cases.[18]

To illustrate FIRRM with patent expiration, suppose an infringing project generates cash flows for ten periods and that the patent expires at the end of period 2. There is an initial investment of $143 million, followed by two cash flows of $50 million and eight cash flows of $33.3 million (i.e., the period cash flow drops by one-third after patent expiration). The cost of capital is 15%, yielding an NPV for the project of $51.4 million. It can be shown that the project has an IRR of 25%, for a spread of 10%.

Suppose first that delay is not an option and that the infringer has no non-infringing alternative. The maximum negotiated royalty therefore would have a present value equal to the entire NPV of the project, or $51.4 million. The running royalty would be assessed only against the first two cash flows. The FIRRM running royalty rate in this scenario is 63.2% of the infringing profits (i.e., the infringer would just earn its cost of capital if it retained 36.8% of the first two cash flows and 100% of the remaining cash flows).[19] If the infringer had an expected 40% profit margin during the infringement period, the royalty as a percentage of revenue would be 25.3% (63.2% times 40%).

Now suppose that the infringer could simply delay the entire project by two years until the patent expired, thereby avoiding all infringement liability. The project would be less valuable in this example for several reasons. The first two cash flows would fall to $33.3 million (the post-expiration level) and each cash flow would be discounted by two additional years. Assume the project still has a useful life of ten years with the delay.[20] This new alternative would have an NPV of $18.4 million (approximately 36% of the original infringing project), compared to a value of zero without the delay option. Accordingly, the maximum

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[18] For example, Judge Tenney included "the duration of the patent and the term of the license" as a Georgia-Pacific factor but he did not provide any analysis of its effect, even though the infringing sales commenced only about four years before expiration of the patent.

[19] The royalty payment each period would be $31.6 million (63.2% times $50 million). The present values when discounted at 15% are $27.5 million and $23.9 million, for a total of $51.4 million. The value of the royalty equals the full NPV of the infringing project because there is no alternative available to the infringer in this example.

[20] The appropriate useful life of the investment might be shorter with delay if there was an externally determined investment horizon, e.g., if it was known that the patented technology would be replaced at a set time in the future. Additional economic analysis would be needed to address this question.

negotiated royalty would fall to $33 million based on the difference in present values. The FIRRM royalty rate on the cash flows under these assumptions drops to 40.6% (or 16.2% of revenue with a 40% profit margin). The delay option leads to a lower royalty.

Tables 3 and 4 present indicative results from the patent expiration analysis. The tables use the same assumptions as Tables 1 and 2. In addition, they incorporate remaining a patent term of two years from the time of the hypothetical negotiation. Cash flows are assumed to decline by 33.3% after expiration. The tables are consistent with the example just discussed (e.g., Table 3c shows 63.2% for the first scenario discussed in this example). The analysis shows that, in principle, the royalty could exceed 100% of the profits in the zero NPV ratio case when the spread is high and the time to expiration is short. However, this is a circumstance where the delay option is likely to have high value, making a high NPV ratio the more relevant assumption with a correspondingly lower royalty.

TABLE 3

FIRRM Indicative Royalties with Patent Expiration
(% of Infringer's Pre-Tax Incremental Profit)

5 Year Life, 2 Years to Expiration

a) 0% NPV Replacement Ratio

IRR Spread	Royalty
0%	0.0%
5%	16.8%
10%	31.0%
15%	43.0%
20%	53.3%
25%	62.2%
100%	121.9%

b) 50.0% NPV Replacement Ratio

IRR Spread	Royalty
0%	0.0%
5%	8.4%
10%	15.5%
15%	21.5%
20%	26.7%
25%	31.1%
100%	61.0%

10 Year Life, 2 Years to Expiration

c) 0% NPV Replacement Ratio **d) 50.0% NPV Replacement Ratio**

IRR Spread	Royalty	IRR Spread	Royalty
0%	0.0%	0%	0.0%
5%	35.9%	5%	17.9%
10%	63.2%	10%	31.6%
15%	84.5%	15%	42.2%
20%	101.4%	20%	50.7%
25%	115.1%	25%	57.5%
100%	189.5%	100%	94.8%

Note: based on 15% cost of capital

TABLE 4

FIRRM Indicative Royalties with Patent Expiration
(% of Infringer's Revenues)

5 Year Life, 2 Years to Expiration

a) 0% NPV Replacement Ratio **b) 50.0% NPV Replacement Ratio**

IRR Spread	Royalty with Infringer's Incremental Profit Margin @ 20%	30%	40%	50%	IRR Spread	Royalty with Infringer's Incremental Profit Margin @ 20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%	0%	0.0%	0.0%	0.0%	0.0%
5%	3.4%	5.1%	6.7%	8.4%	5%	1.7%	2.5%	3.4%	4.2%
10%	6.2%	9.3%	12.4%	15.5%	10%	3.1%	4.6%	6.2%	7.7%
15%	8.6%	12.9%	17.2%	21.5%	15%	4.3%	6.5%	8.6%	10.8%
20%	10.7%	16.0%	21.3%	26.7%	20%	5.3%	8.0%	10.7%	13.3%
25%	12.4%	18.7%	24.9%	31.1%	25%	6.2%	9.3%	12.4%	15.6%
100%	24.4%	36.6%	48.8%	61.0%	100%	12.2%	18.3%	24.4%	30.5%

10 Year Life, 2 Years to Expiration

c) 0% NPV Replacement Ratio

IRR Spread	Royalty with Infringer's Incremental Profit Margin @ 20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	7.2%	10.8%	14.4%	17.9%
10%	12.6%	19.0%	25.3%	31.6%
15%	16.9%	25.3%	33.8%	42.2%
20%	20.3%	30.4%	40.5%	50.7%
25%	23.0%	34.5%	46.0%	57.5%
100%	37.9%	56.9%	75.8%	94.8%

d) 50.0% NPV Replacement Ratio

IRR Spread	Royalty with Infringer's Incremental Profit Margin @ 20%	30%	40%	50%
0%	0.0%	0.0%	0.0%	0.0%
5%	3.6%	5.4%	7.2%	9.0%
10%	6.3%	9.5%	12.6%	15.8%
15%	8.4%	12.7%	16.9%	21.1%
20%	10.1%	15.2%	20.3%	25.3%
25%	11.5%	17.3%	23.0%	28.8%
100%	19.0%	28.4%	37.9%	47.4%

Note: based on 15% cost of capital

Royalty rates with patent expiration can be higher compared to the earlier analysis even though the total present value of the royalty is lower. To understand this result, recall that the royalty rate functions to generate a given present value for the total stream of royalty payments. When the patent expires during the life of the project then there are fewer cash flows available to yield the present value, implying that a higher rate is necessary. For example, Table 2a indicates that a five year life, zero NPV alternative, 30% profit margin, 10% spread, and 15% cost of capital implies a FIRRM royalty rate equal to 5.9% of each sales each period. Table 4a considers the same parameters but with expiration after two years and a 33.3% reduction in cash flows after expiration. The FIRRM royalty rate rises to 9.3% but is applied only to the first two periods instead of all five.[21]

4. COMPARISON OF FIRRM TO OTHER METHODS

A. THE GEORGIA-PACIFIC FACTORS

Broadly speaking, the Georgia-Pacific factors focus on royalties for comparable licenses, the profitability of the patented invention, and the potential for lost sales by the patent holder. They also include expert

[21] This comparison is only intended to be illustrative. In practice, it may not be reasonable to assume the same IRR spread in the case of expiration since the infringer's cash flows are assumed to decline. A smaller spread would reduce the royalty.

opinion, which is a factor in the sense that it can be based on any other arguably relevant consideration, and reconstruction of a hypothetical negotiation, although the court pointed out that the hypothetical negotiation is "more a statement of approach than a tool of analysis." The factors in the decision are grouped in these categories as follows.

Factors Based on Comparable Royalties

The royalties received by the patent holder for the licensing of the patent in suit, proving or tending to prove an established royalty.

The rates paid by the licensee for the used of other patents comparable to the patent in suit.

The nature and scope of the license, as exclusive or non-exclusive; or as restricted or non-restricted in terms of territory or with respect to whom the manufactured product may be sold.

The duration of the patent and the term of the license.

The portion of the selling price or profit that may be customary in the particular business or in comparable businesses to allow for use of the invention or analogous inventions.

Factors Based on Profitability of the Patented Invention

The established profitability of the product made under the patent; its commercial success; and its current popularity.

The utility and advantages of the patent property over the old modes or devices, if any, that had been used for working out similar results.

The nature of the patented invention; the character of the commercial embodiment of it as owned and produced by the licensor; and the benefits to those who have used the invention.

The extent to which the infringer has made use of the invention; and any evidence probative of the value of that use.

The portion of the realizable profit that should be credited to the invention as distinguished from non-patented elements, the manufacturing process, business risks, or significant features or improvements added by the infringer.

Factors Based on Potential Loss of Sales by the Patent Holder

The commercial relationship between the licensor and licensee, such as, whether they are competitors in the same territory in the same line of business; or whether they are inventor and promoter.

The licensor's established policy and marketing program to maintain his patent monopoly by not licensing others to use the invention or by granting licenses under special conditions designed to preserve that monopoly.

The effect of selling the patented product in promoting sales of other products of the licensee; the existing value of the invention to the licensor as a generator of sales of his non-patented items; and the extent of such derivative or convoyed sales.

Expert Opinion and the Hypothetical Negotiation

The opinion testimony of qualified experts.

The amount that a licensor (such as the patentee) and a licensee (such as the infringer) would have agreed upon (at the time the infringement began) if both had been reasonably and voluntarily trying to reach an agreement; that is, the amount which a prudent licensee—who desired, as a business proposition, to obtain a license to manufacture and sell a particular article embodying the patented invention—would have been willing to pay as a royalty and yet be able to make a reasonable profit and which amount would have been acceptable by a prudent patentee who was willing to grant a license.

The Georgia-Pacific factors do not prescribe any particular method for quantifying the appropriate royalty. They do imply that the available market royalty rates are likely to require extensive adjustment before they can be considered fairly comparable, taking account of such issues as differing degrees of licensing exclusivity, duration, field of use, and potential overlap and competition with the patent holder's own sales.

The language referring to "utility and advantages of the patent property over the old modes or devices" does not provide a great deal of direction but it is consistent with a focus on the NPV difference as the basis for the royalty. From an economic point of view, the amount that should be credited to "non-patented elements, the manufacturing process, business risks, or significant features or improvements added by the infringer" should be grounded in the patent holder's cost of capital, again indicating the usefulness of FIRRM in quantifying the analysis.

Finally, the potential loss of sales by the patent holder should be reflected in the expected cash flows and rates of return that underlie FIRRM. However, the patent holder's expected loss does not necessarily translate dollar for dollar to the infringer's gain. Large expected losses may not be fully recoverable through a royalty that is required to leave the infringer with at least a zero NPV investment.

B. COMPARABLE ROYALTY ANALYSIS

The Georgia-Pacific factors based on comparable royalties underscore the pitfalls of using databases of publicly disclosed royalty rates to derive a reasonable royalty. Indeed, there are many additional factors that can be significant when assessing comparability of rates. Royalties may be part of a larger transaction in IP that includes cross-licenses. In addition, parties may contribute know-how and product support as non-cash features of the deal, further clouding the comparability of a given royalty to measure the stand-alone value of a patent. There are also potentially important considerations when projects require licensing of a portfolio of IP. The total royalty resulting from piecemeal adding up of royalties from a database may significantly exceed the level that is consistent with keeping the project return at least as high as the next best alternative.

Royalty databases generally contain a wide range of rates within an industry.[22] There is no compelling reason to select the average or median value since it may not reflect the economic circumstances of either the infringer or the patent holder. FIRRM shows that a wide range of rates can arise due to factors such as the range of investment alternatives that were available to the licensee; such alternatives may either be unidentified in the database or irrelevant to the infringer. Indeed, the research and analysis needed for a principled choice of any value from the database is likely to be similar to the effort required to carry out the more rigorous investment analysis approach developed in this article.

C. THE 25% AND 5% RULES

Two different conventions are sometimes used as shortcuts to avoid a more thorough analysis of a reasonable royalty. The "25% rule" assigns the licensor a royalty equal to 25% of the infringer's pretax gross profit.[23] One problem with the 25% rule surrounds the appropriate definition of

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[22] See, e.g., Daniel McGavock, David Haas, and Michael Patin, "Licensing Practices, Business Strategy, and Factors Affecting Royalty Rates: Results of a Survey," *Licensing Law and Business Report*, 13 No. 6 (March-April 1991), 205-216 and Degnan & Horton, "A Survey of Licensed Royalties," *les Nouvelles*, June 1997.

[23] See Russell L. Parr, Intellectual Property Infringement Damages at 169.

profit. The most relevant economic measure is the incremental pre-tax cash flow. However, this information is generally not available directly from financial statements. In particular, accounting gross margins on financial statements can include fixed manufacturing costs and exclude variable and marketing costs.[24] As a result, gross margins may misstate the relevant cash flows. As in any other damages analysis, using improper accounting information can introduce errors into the analysis, resulting in royalty rates that are either too high or too low.

More fundamentally, however, the 25% rule assigns royalties without regard to any of the determinants of a reasonable royalty that are integrated in the FIRRM analysis. The resulting royalties can be either excessive or too small, depending on the underlying economics. For example, consider Table 1 with a five-year life, a 15% IRR spread, and a 50% NPV replacement ratio. The largest royalty on profits that could be paid in this example that still allows the infringing firm to earn its cost of capital is 13.7%. Applying the 25% rule in this situation would result in a large loss for the infringer. In contrast, Table 1 also shows that a 15-year investment with a 15% spread and a 0% NPV replacement ratio yields a royalty of 44.1%, considerably higher than the 25% rule.

The "5% rule" is open to similar criticism. This approach simply sets the royalty at 5% of the infringer's sales. However, Table 2 shows that the reasonable royalty for the 5-year, 15% spread, 50% NPV replacement ratio ranges from 2.7% of sales to 6.8%, depending on the infringer's profit margin. Hence the 5% rule can be too high (by a factor of almost 2!) or too low. For the 15-year investment just discussed, the royalty in Table 2 ranges from 8.8% to 22.1% of sales and could be higher if the infringer's profit margin exceeded the levels illustrated in the table.

This analysis also indicates that the 25% rule and the 5% rule in general are not mutually compatible. They only yield equivalent results when the infringer's profit margin is 20%.[25] The 25% rule generates a higher royalty if the profit margin is above this level, otherwise the 5% rule is higher. The rules at best provide a rough benchmark for damages

[24] Michael W. Maher, Clyde P. Stickney, Roman L. Weil, and Sidney Davidson, *Managerial Accounting* at 39–43.

[25] Suppose revenue was $100 and profits were $20 (i.e., 20%). The 5% of sales royalty would yield $5, as would 25% of $20. This equivalence would not hold for a profit rate other than 20%.

but even on this basis they do not provide any guidance for choosing between them when they diverge.

The two rules of thumb are best understood as special cases of FIRRM that may be appropriate to a given situation only by chance. There is always a combination of investment life, cost of capital, IRR spread, and NPV replacement ratio that can generate either a 25% of profits royalty or a 5% of sales royalty, as demonstrated in Tables 1 and 2. However, these values would emerge from a proper analysis only by happenstance. The more reliable procedure would be to determine appropriate values for the inputs to the analysis and to let the economic relationships in the model determine the resulting royalty.

5. TWO CASE STUDIES

A. GEORGIA-PACIFIC V. U.S. PLYWOOD

This case involved infringement by Georgia-Pacific ("GP") of the U.S. Plywood ("USP") patent for deeply striated plywood panels. The patented striation solved problems of surface checking and edge separation. The court described the commercial success of USP's Weldtex panels as "amazing," with total sales in the United States exceeding $56 million over the period 1940–1956. The infringement commenced approximately four years before patent expiration and lasted from March 1955 through September 1958.[26]

Georgia-Pacific provides a remarkable demonstration of the range of economic issues that can arise in a complex royalty case and reveals many analytical pitfalls. A royalty analysis using FIRRM will be presented first, followed by discussion of the findings in the district court opinion and the appeal. Enough information is presented in the various decisions to make this analysis feasible but assumptions have to be made at certain points in the absence of specific data. Indeed, FIRRM is useful for identifying the set of information required for a royalty analysis and for making it easy to assess the effect of different assumptions.

The FIRRM analysis stands in stark contrast to the lengthy damages phase of the case. The cost of capital should be readily available; a value of 15% pre-tax is representative for many companies and will be used

[26] The patent at issue expired on June 9, 1959. See *Georgia-Pacific*, 318 F. Supp. 1116, 1126.

here. GP's expected rate of profit is stated to be at least $48.64 per thousand square feet and its average actual selling price was $159.41, suggesting an expected profit margin of approximately 30% (48.64 divided by 159.41, rounded).[27] The USP patent is assumed to expire in four years, after which GP's profit margin is assumed to fall by 10 percentage points (i.e., by about one-third). There is no specific information on the expected useful life of GP's investment at the time of the hypothetical negotiation. GP argued that Weldtex was facing declining demand, although the court was skeptical.[28] For present purposes, a remaining economic life of 10 years will be assumed.

The two remaining necessary parameters are more difficult, given the state of the record. First is the appropriate IRR spread. There is evidence that the spread could be quite high. The record indicates that GP could enter into production of the infringing product "with only the investment required for a striating machine, amounting to approximately $12,000 to $15,000."[29] GP's actual profits from the infringing sales appear to have been on the order of $200,000 per year.[30] This investment implies an IRR spread in excess of 100%, even allowing for reasonable additional upfront expenses for such items as product testing, marketing, and distribution. Nevertheless, to be conservative, a spread of 100% will be used.

Finally, there is the question of the NPV replacement ratio. The record provides evidence that GP was in a competitive industry.[31] As a large public company, it may be reasonable to assume that GP was already undertaking all available projects with positive expected NPV. Nonetheless, it could be an error to infer an NPV replacement ratio of zero. Assuming GP had the option to postpone its investment in striated plywood production until patent expiration, the NPV replacement ratio for the delayed project might be on the order of 41% (based on a four year delay with a cost of capital of 15%).

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[27] *Georgia-Pacific*, 318 F.Supp. 1116, 1131, 1143.

[28] *Georgia-Pacific*, 318 F.Supp. 1116, 1125, 1126.

[29] *Georgia-Pacific* 318 F.Supp. 1116, 1127.

[30] Based on total infringing revenue of $2.5 million over March 1955 to September 1958 (*Georgia-Pacific*, 318 F.Supp. 1116, 1123) and an assumed profit margin of 30%.

[31] "While one would be more likely, if he wanted a striated panel, to choose between Weldtex and GP striated plywood, he would not necessarily do so in the light of the availability of other decorative panels competitively priced." See *Georgia-Pacific Corp. v. United States Plywood Corp*., 243 F. Supp. 500, 510.

Putting these pieces together, FIRRM implies a running royalty on GP's infringing sales equal to 21.2%. In dollar terms, with an average selling price of $159 per thousand square feet the royalty would be $33.80 (21.2% times 159). The key point is that FIRRM allows the effect of different inputs to be evaluated in a consistent manner. For example, a replacement ratio of 0% with no change in the other parameters yields a royalty of 36.2%, or $57.51. The other parameters can be subjected to similar sensitivity tests.

The district court royalty was $50 per thousand square feet, or 31.4% of the infringing sales. According to FIRRM, GP would still more than make its cost of capital with this royalty (the royalty could be as high as $57.51 under the assumptions made here). In this sense, there is support for the district court's view that the $50 royalty would have enabled GP to realize a reasonable profit.[32]

The appellate court's criticism that GP could not make a profit at the $50 rate is therefore probably misplaced. It likely was not necessary to lower the award to $35.65 if GP's profitability (relative to a zero NPV alternative) was the only consideration. However, in what is almost surely a coincidence, the appellate outcome appears to be roughly consistent with an award based on the infringer's best NPV alternative. At bottom, neither decision offers a satisfactory analysis of the relationship between the royalty and the infringer's profitability. In the context of a reasonable royalty, profitability is properly measured as the expected rate of return on invested capital, not the rate of return on sales. Moreover, the courts did not focus on the small required investment or on the fact that the investment was long-lived but would only pay royalties for a fraction of that period.

Our analysis highlights the economic importance of the NPV of the alternative project. The district court decision is roughly consistent with a view that GP's alternatives had zero NPV. If a 41% replacement ratio is reasonable, as suggested by the delay option, the $33.80 maximum rate from the hypothetical negotiation would be slightly lower than the appellate award. Per *Panduit*, there can be justification for setting the reasonable royalty higher than the outcome of an ex ante negotiation in order to deter infringement (although the cases are unclear whether the royalty should be so high as to depress the return below the cost of

[32] *Georgia-Pacific*, 318 F.Supp. 1116, 1143.

capital in the absence of punitive damages). On this basis there is support for both royalties and FIRRM helps to illuminate what assumptions are required to prefer one over the other. More careful review of the infringer's documents would perhaps have revealed which NPV assumption was most appropriate.

B. *SMITHKLINE DIAGNOSTICS, INC., V. HELENA LABORATORIES CORP.*[33]

Helena infringed a patent owned by SmithKline Diagnostics ("SKD") for a specimen test slide and method for detecting occult (hidden or invisible) blood in fecal matter. The gulf separating the royalties proffered by the parties in this case was even larger than in *Georgia-Pacific*. SKD demanded 48% of the infringing sales while Helena claimed that only a 3% royalty was justified. The district court awarded a 25% royalty and was affirmed on appeal.

The economic information in the opinions is sketchy but enough for FIRRM to yield insights if certain additional assumptions are made. As in the earlier examples, it will be assumed that the infringer's cost of capital was 15%. The patent issued in 1982 and the infringing sales commenced in 1983. The patent will be assumed to have an expected useful life of 10 years at the time of the hypothetical negotiation in 1982. Helena, which apparently was a discount supplier that operated in competitive markets, will be assumed to have a 0% NPV replacement ratio.

The remaining parameter needed for FIRRM is the IRR spread. The spread results from the assumed cash flows over the life of the infringing project and the amount of the initial investment. The annual cash flow due to the infringing sales appears to have been on the order of $300,000, with a profit margin as high as 45.6%.[34] The upfront sunk

[33] SmithKline Diagnostics, Inc. v. Helena Laboratories Corp., 1989 U.S. Dist. LEXIS 14761; 12 U.S.P.Q.2D (BNA) 1375.

[34] Information in the opinion indicates that Helena had average annual sales of $718,000 in the first two years of the infringement and a maximum profit margin of 45.6%, which yields approximate annual profits of $300,000. The maximum Helena profit margin is estimated as follows. SKD claimed total lost profits of $1.51 million. Helena had total sales of $1.58 million and the decision reported that SKD's prices were approximately 50% higher, implying claimed lost revenue of $2.37 million for SKD (1.5 times 1.58). SKD's implied costs would be $860,000 ($2.37 million minus $1.51 million). Assuming this level of costs for Helena implies a profit margin of 45.6% (1 minus 0.86 divided by 1.58). Rounding Helena's estimated annual profit down to $300,000 (45.6% times $718,000, rounded) helps take account of the finding in the opinion that SKD's costs were understated.

investment is not reported. Two scenarios are considered as sensitivity tests. If the investment were equal to one year's estimated cash flow of $300,000, the spread would be 85%. If the investment were equal to one year's estimated revenue of $718,000, the spread would be 25%.

Using FIRRM with an 85% spread and the other parameters implies a maximum running royalty equal to 36.5% of sales. The same analysis with a 25% spread results in a 23.7% royalty. The actual award of a 25% royalty is within this range. The infringer's proposed 3% royalty would require a marked difference in assumptions, such as a higher initial investment that would nearly eliminate any expected return above the cost of capital. FIRRM also indicates what would be necessary to justify SKD's proposed royalty of 48%. The IRR spread would have to be over 1000% to generate such a high royalty in this case. Alternatively, the model indicates that Helena's profit rate would have to be higher than the assumed level of 45.6%, which may not comport with other findings in the case. On balance, FIRRM suggests that the 25% royalty was an economically defensible outcome.

6. CONCLUSION

Economics and basic principles of investment analysis from corporate finance provide a tractable framework that can significantly improve the analysis of reasonable royalties in patent infringement. The FIRRM model described in this article yields a running royalty that reflects that maximum that a licensee would be willing to pay and yet be able to make a reasonable profit. FIRRM incorporates the key insights from the Georgia-Pacific factors and builds on the "analytical approach" of profitability introduced in *TWM* v. *Dura*.[35] It is based on a simplified but flexible discounted cash flow analysis. The basic approach requires only information on the infringer's cost of capital, the internal rate of return and useful life of the infringing investment, and the value of the next best investment alternative available to the infringer. It is easily extended to cover the case of patent expiration during the life of the investment and other important economic features of the infringement.

Using FIRRM is a fact-intensive analysis. The contribution of the model is to identify the most relevant data for assessing royalties and to

[35] TWM Mfg. Co., Inc., v. Dura Corp. & Kidde, Inc. 789 F.2d 895, 229 U.S.P.Q. (BNA) 525.

provide a structure for an economically consistent interpretation of the data. This sort of model-based approach, well-established in antitrust and securities litigation, provides many insights into the issues raised in the patent damages area. FIRRM can be further refined but the model as presented in this article already offers a principled and reasonably transparent way to assess competing royalty claims.

FIRRM also points out pitfalls in the use of methods to determine a royalty that are not based on financial analysis. In particular, approaches such as the 5% and 25% rules are likely to be poor approximations to an appropriate royalty in many cases. These rules are also generally mutually inconsistent. Moreover, royalty databases either may not provide comparable rates or may provide such a wide range of rates due to different economics of the underlying deals that there is little indication of what would be appropriate for the case at issue.

The quality of the economic reasoning in reasonable royalty cases has been improving steadily in the 30 years since *Georgia-Pacific*. FIRRM builds on these developments by using standard methods of investment analysis that have not been widely applied in the cases to date. Ultimately, the model should make it easier to reconcile the wide differences between the royalties argued by the parties and lead to more efficient resolution of patent damages cases.

APPENDIX

The maximum royalty that would be economic for the infringer to pay depends on the cost of capital, the excess return of the infringing project, the net present value of the best available non-infringing alternative, the useful life, and the infringer's profit margin.

A. NOTATION

1. Initial investment is a sunk cost F.
2. The infringing project generates constant cash revenue R per year and has a useful life of T years.
3. The infringing project has profit rate μ in the absence of a royalty.
4. The infringing project has profit rate μ_2 with a royalty. The royalty ρ as a percent of sales equals $\mu - \mu_2$.
5. The infringer has cost of capital r. The infringing project has internal rate of return r^* and IRR spread $\chi = r^* - r$.
6. The infringer's best non-infringing alternative project has a net present value equal to γ times the NPV of the infringing project, $0 \leq \gamma \leq 1$.
7. A.F.(r, T) is the annuity factor corresponding to the annual interest rate r and T years. It equals the present value of an annual payment of $1 per year for T years. Mathematically, $\text{A.F.}(r, T) = 1/r\,[1 - 1/(1+r)^T]$.

B. THE FIRRM ANNUITY MODEL

$$\text{NPV(Infringing project)} = \text{NPV(I)} = -F + \mu R\ \text{A.F.}(r,T) \qquad (1)$$

$$\text{IRR(Infringing project)} = r^* = r + \chi$$

$$\Rightarrow F/R = \mu\,\text{A.F.}(r^*,T) \qquad (2)$$

$$\text{NPV(Alternative project)} = \gamma\text{NPV(I)}. \qquad (3)$$

The patent holder can charge a maximum royalty such that the infringer is indifferent between the infringing and non-infringing projects, i.e.,

$$-F + \mu_2 R \text{ A.F.}(r,\text{T}) = \gamma\text{NPV(I)} . \qquad (4)$$

Subtracting (4) from (1) yields

$$(\mu - \mu_2)R \text{ A.F.}(r,\text{T}) = (1 - \gamma)\text{NPV(I)} .$$

Rearranging,

$$\rho = (\mu - \mu_2) = (1 - \gamma) [-F + \mu R\text{A.F.}(r,\text{T})]/[R \text{ A.F.}(r,\text{T})] .$$

$$\Rightarrow \rho = (1 - \gamma) [-F/(R \text{ A.F.}(r,\text{T})) + \mu] .$$

Substituting (2) yields the running royalty

$$\rho = (1 - \gamma) \mu [1 - \text{A.F.}(r+z,\text{T})/\text{A.F.}(r,\text{T})]$$

C. THE FIRRM ANNUITY MODEL WITH PATENT EXPIRATION

The investment has a useful life of T years but the patent expires in T_1 years, with $T_1 < T$. At expiration, margins drop by δ percent due to entry and increased competition. The running royalty is collected only on sales during the patent period.

NPV(Infringing project) = NPV(I)

$$= -F + \mu R \text{ A.F.}(r,\text{T}_1) + (1 - \delta)\mu R \text{ A.F.}(r,\text{T}-\text{T}_1)/(1+r)^{\text{T1}}$$

IRR(Infringing project) = $r^* = r + z$

$$\Rightarrow F/R = \mu\text{A.F.}(r^*,\text{T}) + (1 - \delta)\mu \text{ A.F.}(r^*,\text{T}-\text{T}_1)/(1+r^*)^{\text{T1}}$$

NPV(Alternative project) = γNPV(I).

Equating the NPV of the infringing project with the royalty to the alternative project:

$$-F + \mu_2 R \text{ A.F.}(r,T_1) + (1 - \delta)\mu R \text{ A.F.}(r,T-T_1)/(1+r)^{T1} = \gamma \text{NPV(I)}.$$

Subtracting as before,

$$(\mu - \mu_2) R \text{ A.F.}(r, T_1) = (1 - \gamma)\text{NPV(I)}$$

and rearranging yields

$$\rho = (1 - \gamma)\mu \left[(\text{A.F.}(r,T_1) - \text{A.F.}(r^*,T_1)) + (1 - \delta) (\text{A.F.}(r, T-T_1) / (1+r)^{T1} - \right.$$

$$\left. \text{A.F.}(r^*, T-T_1) /(1+r^*)^{T1})\right] / \text{ A.F.}(r, T_1).$$

When there is no patent expiration during the useful life of the investment then $T = T_1$ and the model reduces to the formulation in section B.